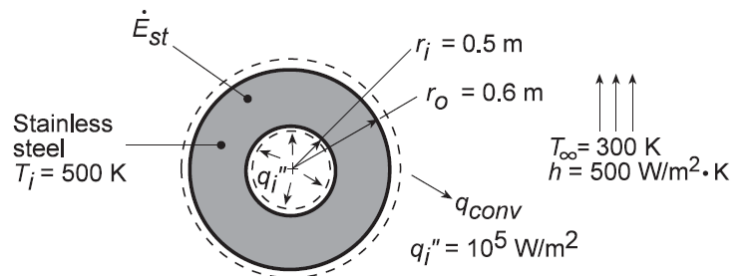


PROBLEM 1.50

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time. Initial rate of temperature change, (b) Steady-state wall temperature, (c) Effect of convection coefficient on canister temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

PROPERTIES: Table A.1, Stainless Steel, AISI 302: $\rho = 8055 \text{ kg/m}^3$, $c_p = 535 \text{ J/kg}\cdot\text{K}$.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \dot{E}_{\text{st}}$. Identifying relevant processes and solving for dT/dt ,

$$q_i''(4\pi r_i^2) - h(4\pi r_o^2)(T - T_\infty) = \rho \frac{4}{3}\pi(r_o^3 - r_i^3)c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p (r_o^3 - r_i^3)} [q_i'' r_i^2 - h r_o^2 (T - T_\infty)].$$

Substituting numerical values for the initial condition, find

$$\left. \frac{dT}{dt} \right|_i = \frac{3 \left[10^5 \frac{\text{W}}{\text{m}^2} (0.5\text{m})^2 - 500 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} (0.6\text{m})^2 (500 - 300) \text{K} \right]}{8055 \frac{\text{kg}}{\text{m}^3} 535 \frac{\text{J}}{\text{kg} \cdot \text{K}} [(0.6)^3 - (0.5)^3] \text{m}^3}$$

$$\left. \frac{dT}{dt} \right|_i = -0.084 \text{ K/s.} \quad <$$

(b) Under steady-state conditions with $\dot{E}_{\text{st}} = 0$, it follows that

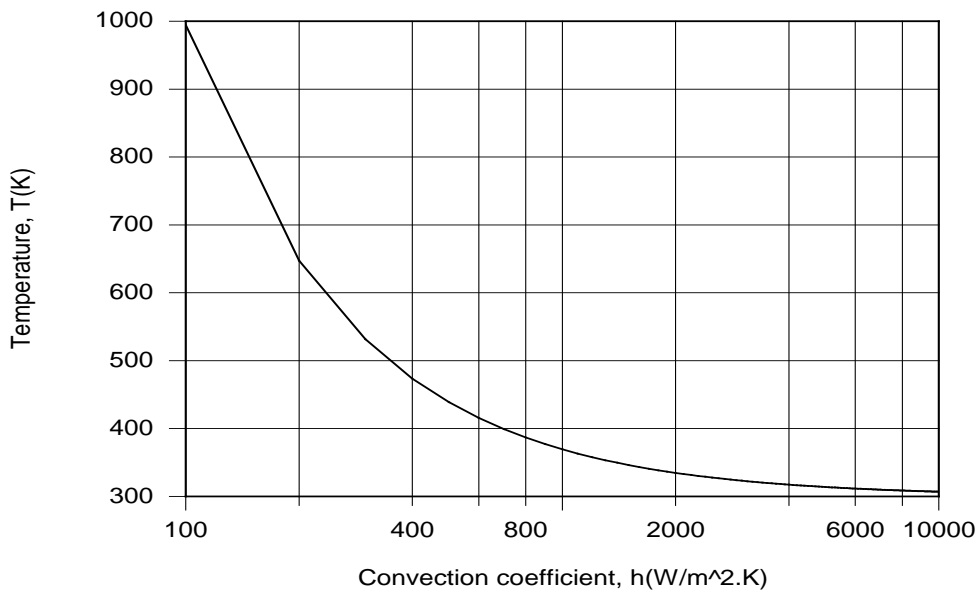
$$q_i''(4\pi r_i^2) = h(4\pi r_o^2)(T - T_\infty)$$

$$T = T_\infty + \frac{q_i''}{h} \left(\frac{r_i}{r_o} \right)^2 = 300\text{K} + \frac{10^5 \text{ W/m}^2}{500 \text{ W/m}^2 \cdot \text{K}} \left(\frac{0.5\text{m}}{0.6\text{m}} \right)^2 = 439\text{K} \quad <$$

Continued

PROBLEM 1.50 (Cont.)

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Hollow Sphere*. As shown below, there is a sharp increase in temperature with decreasing values of $h < 1000 \text{ W/m}^2\cdot\text{K}$. For $T > 380 \text{ K}$, boiling will occur at the canister surface, and for $T > 410 \text{ K}$ a condition known as film boiling (Chapter 10) will occur. The condition corresponds to a precipitous reduction in h and increase in T .



Although the canister remains well below the melting point of stainless steel for $h = 100 \text{ W/m}^2\cdot\text{K}$, boiling should be avoided, in which case the convection coefficient should be maintained at $h > 1000 \text{ W/m}^2\cdot\text{K}$.

COMMENTS: The governing equation of part (a) is a first order, nonhomogenous differential equation with constant coefficients. Its solution is $\theta = (S/R)(1 - e^{-Rt}) + \theta_i e^{-Rt}$, where $\theta \equiv T - T_\infty$,

$S \equiv 3q_i'' r_i^2 / \rho c_p (r_o^3 - r_i^3)$, $R = 3hr_o^2 / \rho c_p (r_o^3 - r_i^3)$. Note results for $t \rightarrow \infty$ and for $S = 0$.